Reef Metrics

We analyze a virtual reef following several routes through the reef dissection map. First, break the reef into panels with trapezoidal faces.

Dissection of a Virtual Reef
Virtual Reefs
Panels **Half panels** **Keepsake**
3D **Three Views** **Loving Reef**
Trapezoids / Trapezia

Second, see the reef (without holes) as the difference between two frusta, and a frustum as the difference between two pyramids.

Wiki Frusta **Rider's Frusta**
Regular Pyramids

Third, see the reef from the top view entirely as a cycle of four lamp shades: the outer lateral surface, the planar top surface, the inner lateral surface, and the planar btm surface.

Lamp shades / Annuli
Lateral faces

Fourth, see the reef as a cycle of similar regular polygons: the outer btm, the outer mdl, the outer top, the center top, the inner top, inner mdl, the hidden inner btm, and the center btm.

Parametric Ratios
Regular Polygons

Fifth, examine radial cross sections of the reef for vertical height, normal thicknesses and holes.

Radial cross section
Cylindrical holes

Sixth, several possibilities are deferred. I list them here to avoid losing them.

Details
Buoyancy

3D
Javascript
FabLab
New shapes
Trisector

Reef Metrics

The gold cells name basis values.

The light green cells refer to other cells, possibly with a mode conversion.

The lilac cells are calculated by proportionality of similar shapes.

The blue cells are calculated by weighted sums, such as differences or averages.

The light yellow cells are calculated by solving right triangles.

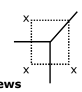
Experiments

This section organizes several experiments as a table, one experiment per column. The slide bar at the top selects which experiment is shown. The results are tabulated for all the experiments. Experiments named in red are under construction. Results with values in red are questionable.

		name	formula		Your Reef	Benchmark	Living Reef (shown)	Keepsake	Loving Reef	Pentagonal	OES Reef	Beer Cooler	Dog Bowl
1	**Results**		data =										
2	number of panels:	n =		6	5	4	6	6	4	5	2.5	8	7
3	polygon interior angle:			60°	72°	90°	60°	60°	90°	72°	144°	45°	51.42857°
4	slant angle:			64.34109°	69.87°	90.00°	64.34°	30.00°	75.52°	69.87°	83.00°	52.88°	67.09°
5	vertical height:	Δh =		1 313/889	1.408	1.000	1.352	0.167	1.452	1.408	0.993	0.797	0.614
6	max radius:	R =	max(BoR,BiR,...) =	1 1/2	1.276	0.707	1.500	0.563	1.061	1.276	0.789	1.089	1.152
7	btm outer exradius:	BeR =		1 1/2	1.276	0.707	1.500	0.563	1.061	1.276	0.789	1.089	1.152
8	btm inner exradius:	BiR =		1 109/947	0.864	0.000	1.115	0.430	0.589	0.864	0.519	0.345	0.320
9	top outer exradius:	TeR =		3/4	0.638	0.707	0.750	0.229	0.530	0.638	0.394	0.436	0.864
10	top inner exradius:	TiR =		272/745	0.432	0.000	0.365	0.097	0.059	0.226	0.259	0.345	0.772
11	outer perimeter:	girth =		9	7.500	4.000	9.000	3.375	6.000	7.500	3.750	6.667	7.000
12				ft	ft	ft	ft	ft	ft	ft	ft	ft	ft
13	outer footprint:		btm polygon area	5.85	3.87	1	5.85	0.82	2.25	3.87	0.46	3.35	3.63
14	annular footprint:	afp =	btm − top area	2.62	2.10	1	2.62	0.34	1.56	2.10	0.26	3.02	3.35
15	inner footprint:		top polygon area	3.23	1.77	0	3.23	0.48	0.69	1.77	0.20	0.34	0.28
16	area-unit:			sq ft	sq ft	sq ft	sq ft	sq ft	sq ft	sq ft	sq ft	sq ft	sq ft
17	gross volume:	outer =	outer frustum =	4.611	3.18	1.00	4.61	0.07	1.91	3.18	0.26	1.39	1.72
18	interior volume:	inner =	inner frustum =	2.089	1.46	0.00	2.09	0.03	0.37	1.11	0.11	0.27	0.53
19	net volume:	∂V =	n * ∂V/n =	2.32	1.64	0.86	2.32	0.04	1.44	1.89	0.13	0.63	1.19
20	volume-unit:	L^3 =		cu ft	cu ft	cu ft	cu ft	cu ft	cu ft	cu ft	cu ft	cu ft	cu ft
21	mass:	mass =	∂V * crete =	277.11	246	129	277	4.378	173	284	20	95	10
22	mass-unit:	M =		lbm	lbm	lbm	lbm	lbm	lbm	lbm	lbm	lbm	lbm
23	weight at sea-level:	weight =	mass at sea-level =	277.11	246	129	277	4.38	173	284	20	95	9.526319
24	underwater weight:	uwt =	weight − displaced =	135.09	119.9	62.8	135.1	2.1	84.1	138.4	9.6	46.2	4.6
25	wt-unit:			lb	lb	lb	lb	lb	lb	lb	lb	lb	lb
26													
27													
28	pressure on floor:												
29													
30	**Inputs**	shown											

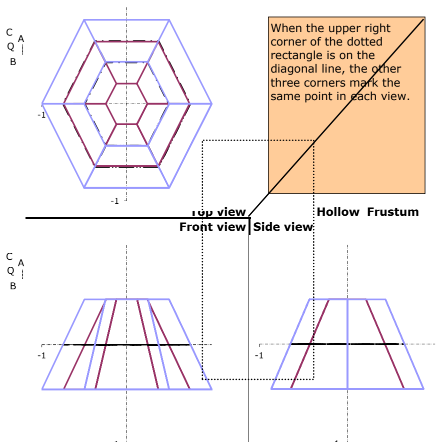
The table outlined with a heavy border below must be cleared before changing its structure. It may be permuted and expanded as desired and later reconstructed using Data>Table....

31	Rob sent measurements for the Loving Reef, except for the btm thickness, which we assume to equals the top thickness.	*Living Reef*	The illegible description of the shown experiment to the right is enlarged on the left.			This column lets you play what-if games, without disrupting other experiments. You can change the table as needed.	This column defines a benchmark that has easily calculated properties. For example, unit outer edges and slant height gives a gross volume of one cubic unit, whatever the system of weights	Rob sent measurements for the Loving Reef, except for the btm thickness, which we assume to equals the top thickness.	Rob sent a Keepsake Reef for other measurements, such as displacement of water. The model is not to scale, but is made of the same material as his other reefs.	We can devise reefs with different configurations. The Loving Reef has a square base.	A Penagonal Reef is an example of a what-if experiment. It is the version used in developing the correct equations as it lacks coincidences.	A fractional number of panels is interesting, but volumes will be misleading.	This experiment shows what happens when the inner walls are vertical. It could be sized to fit the beer can of yuour choice. The holes are horizontal.	This experiment has no holes but has an inverted inner frustum. The material is assumed to be different, such as rubber.
32	panels per reef:	n =	input=	6	panels/reef	5	4	6	6	4	5	2.5	8	7
33	# top holes:	Ktp =	input=	1	holes/panel	1	1	1	1	1	1	1	1	0
34	# mdl holes:	Kmd =	input=	1	holes/panel	1	0	1	1	0	1	1	1	0
35	# btm holes:	Kbt =	input=	2	holes/panel	1	1	2	2	1	2	2	2	0
36	normal holes?	normal? =	input=	TRUE		TRUE	FALSE	TRUE	TRUE	TRUE	TRUE	FALSE	FALSE	FALSE
37	out-unit:	ft =	input=	12	in	12	12	12	12	12	12	12	12	12
38	top hole diameter:	Wtp =	input=	1.5	in	2	2	1.5	0.4	2	1.5	1.5	1.5	0
39	mdl hole diameter:	Wmd =	input=	2.5	in	0		2.5	0.4		2.5	2.5	2.5	0
40	btm hole diameter:	Wbt =	input=	2.5	in	3	3	2.5	0.4	3	2.5	2.5	2.5	0
41	top outer edge:	Te =	input=	9	in	9	12	9	2.75	9	9	9	4	9
42	btm outer edge:	Be =	input=	18	in	18	12	18	6.75	18	18	18	10	12
43	top thickness:	∂Tr =	input=	4	in	2	6	4	1.375	4	4	0.5	1	1
44	btm thickness:	∂Br =	input=	4	in	4	6	4	1.375	4	4	1	8.243	9
45	outer slant height:	Δs =	input=	18	in	18	12	18	4	18	18	12	12	8
46	top holes:	tp =	input=	14	in	15	15	14	3.25	14	14	10	6	0
47	mdl holes:	md =	input=	10	in	6	9	10	2	10	10	6	4	0
48	btm holes:	bt =	input=	4	in	6	6	4	0.625	4	4	3	2	0
49	in-unit	un =	input=	in		in	in	in	in	in	in	in	in	in
50	density:	crete =	guess =	119.688312	pcf	150	150	120	120	120	150	150	150	8
51	density unit:	M/L^3 =	input=	pcf		pcf	pcf	pcf	pcf	pcf	pcf	pcf	pcf	pcf
52	len-unit:	L =	input=	ft		ft	ft	ft	ft	ft	ft	ft	ft	ft
53	area-unit:	L^2 =	input=	sq ft		sq ft	sq ft	sq ft	sq ft	sq ft	sq ft	sq ft	sq ft	sq ft
54	volume-unit:	L^3 =	input=	cu ft		cu ft	cu ft	cu ft	cu ft	cu ft	cu ft	cu ft	cu ft	cu ft
55	mass-unit:	M =	input=	lbm		lbm	lbm	lbm	lbm	lbm	lbm	lbm	lbm	lbm
56	acc-unit:	L/T^2 =	constant=	gee		gee	gee	gee	gee	gee	gee	gee	gee	gee
57	weight unit:	ML/T^2 =	input=	lb		lb	lb	lb	lb	lb	lb	lb	lb	lb
58	pressure unit:	M/LT^2 =	input=	psi		psi	psi	psi	psi	psi	psi	psi	psi	psi



Three Views

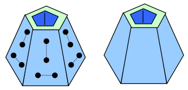
These views of the measured object are constructed for points calculated in the 'Points' worksheet.



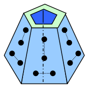
Three Views

0.891304348

Results



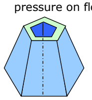
We show the results first. Each result is calculated from partial results below, often in several ways. The top, front and side views are displayed as a sanity check and are not part of the calculations of other results.



A virtual reef is a blank reef with some number of holes in it. These are its net properties.

The mass is volume times density; but the added pressure on the sea floor is underwater weight divided by contact area.

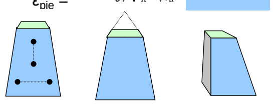
Virtual Reefs	*name*	*formula*	*properties*		*outer*	*inner*
net surface area:	surface =	n * panel=	19.66	sq ft		
net volume:	∂V =	n * ∂V/h =	2.32	cu ft		
mass:	mass =	∂V * crete =	277	lbm		
pressure on floor:	pressure =	uwt / afp =	52	psi		

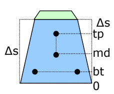


A blank reef has no holes in its panels. These are its gross properties.

Blank reefs	*name*	*formula*	*properties*			*outer*	*inner*	*annular*			
half angle:	ø =	π / n =	30°								
slant angle:		atan Δr/Δh =				64.34°	115.66°				
vertical height:	Δh =	below =	1 313/889	ft						16.2	in
mean normal thickness:	∂t=	below =	195/649	ft						3.6	in
footprint:		below =				5.85	3.23	2.62	sq ft		
total surface area:		n * panel=									
lateral areas:		outer + inner =									
volume:	∂V =	outer − inner =	2.5	cu ft		4.61	2.09		cu ft		
ε_{pie} =		∂V − n * V/h =			√				cu ft		

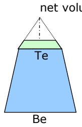
Inputs





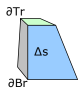
A panel is here defined as one segment of a virtual reef cut radially at each lateral edge. We consider the holes here, including their sizes and placements. We may select whether they are horizontal or normal to the outer lateral surface.

Panels			*relative heights*		*rel widths*	*widths*		*counts*	*widths*	*heights*	
outer slant height:	Δs =	input=	1	Δs		1 1/2	ft			18	in
top holes:	tp =	input=	7/9	Δs	1/12	1/8	ft	1	1.5	14	in
mdl holes:	md =	input=	5/9	Δs	5/36	5/24	ft	1	2.5	10	in
btm holes:	bt=	input=	2/9	Δs	5/36	5/24	ft	2	2.5	4	in
holes:						*holes*		4	holes/panel	TRUE	normal?
number of panels:	n =	input=	*properties*					6	panels/reef		
net surface area:	surface =	areas + holes =	3.28	sq ft		0.48	sq ft				
net volume:	net vol=	vol − holes =	0.39	cu ft		0.03	cu ft				



A blank panel is one panel with no holes or one segment of a blank reef cut radially at each lateral edge. All six faces are trapezoids, but the two cut faces are congruent. All midsections are also trapezoids. The panel's volume is the difference between two pie-shaped frusta, which is like an outer pie crust.

Panels					*lateral faces*	*horizontal faces*	*exposed faces*	*cut radial faces*		*counts*	*widths*	
conversion factor:			1	ft	=						12	in
top outer edge:	Te =	input=	3/4	0.000							9	in
btm outer edge:	Be =	input=	1 1/2	ft							18	in
areas:	A =	face + opposite =			2.80	0.62	3.42	1.04	sq ft			
volume:	V/h =	outer − inner =				0.42			cu ft			



A half panel is here defined as a blank panel divided radially in half. It exposes four more values needed for defining a virtual reef. Density is needed for calculating mass, weight and pressure on the annular footprint. We may use the density of any measured sample, such as a half panel. Cast cylinders of concrete are standard. I suggest filling an 8-ounce paper cup with the material to be measured and recording its weight as a function of drying time. Measure displaced water or top and btm diameters and slant height to get its volume.

The boxed cells are for inputs. You can change them to see what would happen. You can move the ∂Br selector to change the btm thickness method. But move it back for the tabulation.

Half panels

	name	*formula*	*properties*			*other value*	*Hollow frustum*	*Annular frustum*	*Inner prism*	*measured*	*input unit*
btm selector:	move ∂Br between bounds =		[redacted]			∂Br				[redacted]	
btm thickness:	∂Br =	input =	1/3	ft		4	4	8	11 633/797	4	in
top thickness:	∂Tr =	input =	1/3	ft						4	in
density:	crete =	guess =	119 53/77	pcf	from	http://www.engineeringtoolbox.com/concrete-properties-d_1223.html					

The Keepsake Memorial is a small artificial reef for desktop display. It is not to scale but is made of then same material as the underwater memorials. I measured the volume by cutting the top off a gallon milk jug, filling it with water, submersing the model, removing it and measuring how much water it takes to refill the jug, trying not to spill any.

We should use specific gravity (density in units of water) as that is universal, working with all systems of weights and measures.

Keepsake

http://www.livingreefmemorial.com/living-reef/

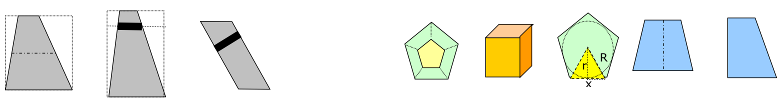
weight:	wt =	bathroom scale =	3 2/5	lb	5/176	cu ft	digital bathroom scale accurate to 0.1 lb.
capacity:	cap =	water held =	3/4	cups	5/798	cu ft	inverted, lined with Saran Wrap and filled with water
volume:	vol=	water displaced =	3 2/5	cups	5/176	cu ft	or about 800 cc in a Pyrex measuring cup
conversion:	cu ft =	in cups =	119 53/77	cups	1	cu ft	https://www.google.com/search?client=gmail&rls=gm&q=cups%20per%20cubic%20foot
density:	crete =	estimate =	2	water	119.69	pcf	estimated, rounded ± 2 pcf
displaced seawater:	seawater =	Google =	1 1/40	water	61.34	pcf	http://www.engineeringtoolbox.com/specific-gravity-liquids-d_336.html
underwater weight:	crete =	crete - seawater =	39/40	water	58.35	pcf	submersed weight is sea-level weight minus displaced weight

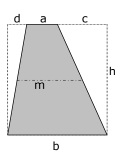
Rob measured the volume of the Loving Reef at 757 ounces. The weight would give us the density as poured. The problem is that volume, weight and density change as the concrete cures. The easiest way to check the cure of a formation is to take a pour sample in a paper cup. Mark the with the date of the pour and identity of the formation. Keep it with the formation so it cures at the same rate. Periodically measure the weight and volume of the sample and record their ratio as the density. The volume could be measured as the weight of water the sample displaces. A dry method is to measure the slant height and both diameters (top and bottom) of the sample so we can calculate its vertical height and volume. I think the formation is cured at peak density, but that should be verified.

Loving Reef

vol= liquid measure = 757 oz 94.625 cups

Surfaces



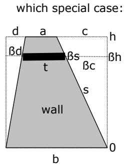


A trapezoid (*Brit.* trapezium) has at least one pair of parallel sides. The other two sides are called legs. We do not assume that the legs are not parallel or do not cross, so degenerate cases are included. The area is the mean width times the height, which will be strange when the legs are crossed because one lobe is a hole with negative area. Don't cross your legs. We assume d+a+c=b, so d<0 means a is undercut.

A hole in the wall is ambiguous: Is it perpendicular to the inner, center, or outer face? Those thicknesses are hard to calculate. It may be horizontal or normal to an outer face.

Trapezoids / Trapezia

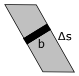
			outer face	*inner face*	*center face*	*top face*	*btm face*	*mdl face*	*inradius cuts*	*cut faces*	*unit*
cap:	a =	given =	3/4	272/745	528/947	272/745	1 109/947	299/404	1/3	311/808	ft
base:	b =	given =	1 1/2	1 109/947	1 167/543	3/4	1 1/2	1 1/8	1/3	311/808	ft
mean width:	m =	(b+a)/2 =	1 1/8	299/404	318/341	528/947	1 167/543	318/341	1/3	311/808	ft
split difference:	Δ =	(b-a)/2 =	3/8	3/8	3/8	51/265	51/265	51/265	0	-0	ft
height:	h =	given =	1 1/2	1 1/2	1 1/2	1/3	1/3	1/3	1 313/889	1 313/889	ft
area:	A =	m * h =	1.69	1.11	1.40	0.19	0.44	0.31	0.45	0.52	sq ft
right offset:	c =	given =	3/8	3/8	3/8	51/265	51/265	51/265	202/311	3/4	ft
left offset:	d =	b-a-c =	3/8	3/8	3/8	51/265	51/265	51/265	- 202/311	- 3/4	ft
which special case:			*trap*	*trap*	*trap*	*trap*	*trap*	*trap*	*para*	*para*	



A hole in the wall is ambiguous: Is it perpendicular to the inner, center, or outer face? Those thicknesses are hard to calculate. If it were a horizontal tunnel, the tunnel length depends on where the hole is. This is complicated and ellipse areas are not correctly calculated.

Tunnels in the walls

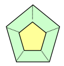
			tp holes	*md holes*	*bt holes*		*unit*
	ßs =	given =	7/9	5/9	2/9	s	
tunnel length:	tl=	b - ß(b-a) =	1/3	1/3	1/3	ft	



If the walls have constant thickness, then the trapezoid is a parallelogram and the normal wall thickness ∂t is unambiguous and easy to calculate as wall area divided by slant height. Otherwise, we may as well use this ratio as the average wall thickness. Furthermore, the walls of the model reef are constant thickness and the holes are normal, not horizontal.

Normal holes

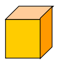
tn =	area / Δs =	195/649		ft



A lamp shade consists of two similar parallel polygons, which may be connected by lateral edges. It is an annulus when the polygons are coplanar.

Lamp shades / Annuli

					annuli			lamp shades			
					top	*btm*	*mdl*	*outer*	*inner*	*center*	*unit*
thickness / slant height:	∂r =	below =		ft	1/3	1/3	1/3	1 1/2			ft
annular area:	area =	outer − inner =		sq ft	1.12	2.62	1.87				sq ft



Geometric quantities are often parametric, scaling linearly with a parameter, its square, or its cube. Setting the parameter to 1 then gives us the scaling factors. This approach is limited to simple structures.

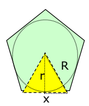
The calculations for r and x below are intermingled because two x's are measured, one or two thicknesses are subtracted from outer r's to get inner r's.

Parametric Ratios

		constant	*formula*	*ratio*	*unit*	visible polygons (measured): *inner top*	*outer top*	*outer btm*	*outer mdl*	hidden polygons: *inner btm*	*inner mdl*	
0	half angle:	ø =	π / n =	355/678	rad		355/678					
0	tangent:	tanø =	tan ø =	571/989	x/2r		571/989					
0	edge/inradius:	2tanø =	2 * tanø =	1 28/181	x/r		1 28/181					
1	inradius:	r =	parameter =	1	r	252/797	202/311	1 93/311	303/311	535/554	407/635	ft
1	given edges:	x =	r * 2tanø =	1 28/181	r	272/745	3/4	1 1/2	1 1/8	1 109/947	299/404	ft
2	inradius squared:	r^2 =	r ^ 2 =	1	r^2	0.10	0.42	1.69	0.95	0.93	0.41	sq ft
3	inradius cubed:	r^3 =	r ^ 3 =	1.0	r^3	0.032	0.274	2.192	0.925	0.901	0.263	cu ft
	standard brick:	ΔhΔrΔx =	Δh * Δr * Δx =		hrx							

Edges are sometimes calculated from inradii; other times, vice versa.

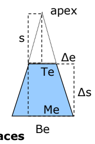
A unit polygon is here defined as an regular polygon with unit edge length. The illustrations show pentagons for generality so that the equations work for any number of sides. Set *n*=4 for degenerate cases to check results. Set n=6 for hexagonal reefs.



The blue row is a check that two ways of calculating the area of a polygon agree. I had found two discrepancies but could not find an error in the alternative method. The bug was a subtle r^2 problem that affected the obvious pie method and all its bounds checks. I found it by ratios.

Regular Polygons

							visible polygons: *inner top*	*outer top*	*outer btm*	*outer mdl*	hidden polygons: *inner btm*	*inner mdl*	
1	perimeter:	nx =	n * x =	6 181/195	r		2 73/383	4 1/2	9	6 3/4	6 404/585	4 382/867	ft
1	semiperimeter:	semi =	nx / 2 =	3 181/390	r		1 73/766	2 1/4	4 1/2	3 3/8	3 202/585	2 191/867	ft
1	exradius:	R =	x / sin ø =	1 28/181	r		272/745	3/4	1 1/2	1 1/8	1 109/947	299/404	ft
2	lemon pie triangle:	pie =	r * x/2 =	0.58	r^2		0.06	0.24	0.97	0.55	0.54	0.24	sq ft
2	inscribed circle:	inscribed =	π * r^2 =	3.14	r^2		0.31	1.33	5.30	2.98	2.93	1.29	sq ft
2	polygon area:	A =	n * pie =	3.46	r^2		0.35	1.46	5.85	3.29	3.23	1.42	sq ft
	discrepancy:	ε =	A − r*semi =		r^2	√							sq ft
2	circumscribed circle:	circum =	π * R^2 =	4.19	r^2		0.42	1.77	7.07	3.98	3.91	1.72	sq ft

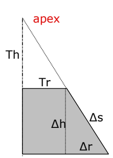


A lateral face lets us locate the apex except when the reef is a hollow prism, in which case the apex is at infinity. We want our calculations to work anyway, so we include it as a special case and expect its simpler calculations to agree with the general calculations.

Lateral faces

	name	*formula*	*outer face*	*inner face*	*unit*
average width:	Me =	(Be+Te)/2 =	1 1/8	299/404	ft
split difference:	Δe =	(Be−Te)/2 =	3/8	3/8	ft
common ratio:	Te/Be =	Te / Be =	1/2	2	
do not locate apex:	s =	Δs / (B − 1) =	1 1/2		
lateral face area:	area =	Δs * Me =	1 11/16	1 51/463	sq ft

The apex may be far away, when Δe≈0.



The radial cross section of a frustum (through the axis) that is perpendicular to a lateral face lets us calculate parameters we wanted in the first place. We get Δh/Δr = Th/Tr or their inverses, so we can locate Th = Tr * Δh/Δr when Δr is large enough.

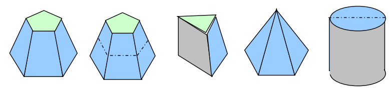
Radial cross section

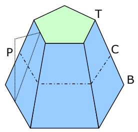
	name	*formula*	*outer frustum*			*inner frustum*	*center slice*
top thickness:	∂Tr =	input =	1/3	ft			
btm thickness:	∂Br =	dr+∂Tr+Δr =	1/3	ft			
slant height:	Δs =	√ $Δh^2+Δr^2$ =	1 1/2	ft	ds =	1 1/2	
offsets:	Δr =	Br − Tr =	202/311	ft	dr =	- 202/311	
vertical height:	Δh =	√ $Δs^2−Δr^2$ =	1 313/889	ft			

top height:	Th =	Tr * Δh/Δr =	1 313/889	ft
top height:	Bh =	Br * Δh/Δr =	2 169/240	ft
btm height:	Bh =	Th + Δh =	2 169/240	ft
average thickness:	μ∂ =	(∂Tr+∂Br)/2 =	1/3	ft
section area:	area =	μ∂ * Δh	32/71	sq ft
mean normal thickness:	∂t=	area / Δs =	195/649	ft

Volumes

A regular right pyramidal frustum is a regular right pyramid with its similar pyramidal top cut off parallel to its base.



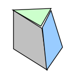


QuickTime™ and a TIFF (LZW) decompressor are needed to see this picture.

A pyramidal frustum (regular or irregular, right or oblique) is a pyramid with its pyramidal top cut off parallel to its base. Its volume ΔV is the difference. Its top and bottom are similar polygons.

The problem is that we have several ways to calculate V/P, but they did not agree. Why not? First, I got it right on the fully measured outer frustum ignoring the special cases. This worked, satisfying all the discrepancy checks.

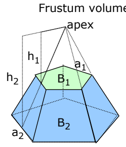
One slice of a frustum is still a frustum, so its volume is easily obtained from its height and horizontal faces.

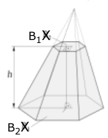


Rider's Frusta

				frustum	frustum	outer pie	inner pie	unit
perpendicular distance:	P =	Δh =		1 313/889				
top area:	T =	below =		1.46	0.35	0.24	0.06	sq ft
bottom area:	B =	below =		5.85	3.23	0.97	0.54	sq ft
middle area:	C =	below =		3.29	1.42	0.55	0.24	sq ft
root mean square:	√BT =	√ B*T =		2.92	1.06	0.49	0.18	sq ft
average of three:	μA =	(B+T+√BT)/3 =		3.41	1.54	0.57	0.26	sq ft
check average of six:	ε_{six} =	μA - (B+T+4C)/6 =	√					sq ft
check derivation:	ε_{two} =	μA - (4C -√BT)/3 =	√					sq ft
Frustum volume:	ΔV =	P * μA =		4.611	2.089	0.768	0.348	cu ft

A regular right pyramidal frustum is a regular right pyramid with its similar pyramidal top cut off parallel to its base.





The Wiki discussion has B_2 on the bottom, unlike the figure.

Wiki Frusta

source: http://en.wikipedia.org/wiki/Frustum

				outer frustum	inner frustum	
perpendicular distance:	h =	Δh =		1 313/889	1 313/889	ft

$$V = \frac{h_2 B_2 - h_1 B_1}{3}$$

The simplest definition of the volume of a frustum of a pyramid is the magnitude of the difference in the volumes of two finite pyramids. It fails for the volume of a prism, which is the difference of the volumes of two infinite pyramids.

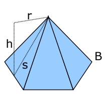
				outer frustum	inner frustum	
top face:	B_1 =	top =		1.46	0.35	sq ft
bottom face:	B_2 =	btm =		5.85	3.23	sq ft
volume:	ΔV =	|btm - top| =		4.38	2.88	cu ft

$$V = \frac{h}{3}(B_1 + B_2 + \sqrt{B_1 B_2})$$

				outer frustum	inner frustum	
root mean square:	$\sqrt{B_1 B_2}$ =	$\sqrt{B_1 * B_2}$ =		2.92	1.06	sq ft
average of three:	V/h =	$(B_1+B_2+\sqrt{B_1B_2})/3$ =		3.65	1.88	sq ft
equivalent equations:	ε_{wiki} =	μA - V/h =	??	-0.244	-0.335	sq ft
volume:	V =	h * V/h =		4.9	2.5	cu ft

$$V = \frac{nh}{12}(a_1^2 + a_2^2 + a_1 a_2)\cot\frac{\pi}{n}$$

				outer frustum	inner frustum	
top edge:	a_1 =	Te =		3/4	272/745	ft
btm edge:	a_2 =	Be =		1 1/2	1 109/947	ft
mean rect:	M sq =	$(a_1^2+a_2^2+a_1a_2)/3$ =		1.31	0.59	sq ft
mean area from edge:	?V/h =	M sq * n / 4tanø =		3.41	1.54	sq ft
discrepancy:	ε_{edges} =	V/h - ?V/h =	??	0.244	0.335	sq ft

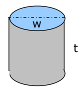


A regular pyramid is a horizontal polygon extended to an apex above the center of the polygon. This approach is faulty when calulating the apex of a prism, which is infinitely far away. This is developed as a means of debugging frustum volume equations.

Regular Pyramids

			ratio		outer pyramids: top	outer pyramids: btm	annular	inner pyramids: inner top	inner pyramids: inner btm	
	B =	above =	3 181/390	r^2	1 293/635	5 674/797		80/231	3 89/386	sq ft
vertical height:	h =	$\sqrt{s^2 - r^2}$ =	1	h	1 93/311	2 311/520				ft

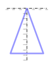
		V/h =	B/3 =	1 28/181	r^2	303/622	1 756/797			80/693	sq ft
	constant:	B/h^2 =		3 181/390		181/209	181/209				
	volume:	V =	h * area/3 =	1 28/181	hr^2	0.6	5.1				cu ft



Cylindrical holes

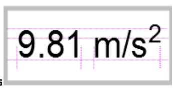
				ratios		tp hole	md hole	bt holes		
1	normal thickness:	t=	parameter=	1	t	195/649	195/649	195/649	ft	
1	widths:	w =	parameter=	1	w	1/8	5/24	5/24	ft	
1	circumference:	C =	π * w =	3.141592654	w	355/904	502/767	502/767	ft	
2	end area:	cap =	$\pi * (w/2)^2$ =	0.785	w^2	0.012	0.034	0.034	sq ft	
1.767	wrap area:	wrap =	C * t=	3.141592654	tw	0.118	0.197	0.197	sq ft	
1.883	net surface area:	dA =	wrap - 2cap =	1.570796327	t^2	0.093447697	0.12847539	0.12847539	sq ft	(at-2bw)w =
3	volume:	plug =	cap * t=	0.785	tw^2	0.003687231	0.01024231	0.01024231	cu ft	

Reef Work Below: Calculations above this marker have been scrutinized.



Details

Some of the parts of a reef could be rendered for better illustration. In particular, three views of a panel with holes would give a better sense of proportion.



SI units

Ensure that metric measurements or calculations produce equivalent results.

Calculate the correct underwater weight of a reef, which is its sea-level weight minus the weight of seawater displaced.

Buoyancy

http://www.goldgold.com/rocks-dont-weigh-as-much-when-submerged-underwater.html

3D

3D Scene not yet ready

3D Stereo not yet ready

I have old spreadsheet logic to display a turning 3D stick figure in stereo motion, but am not installing it here until it helps somehow. Meanwhile, three views of a virtual reef help a lot. The displays are not part of the calculations, just a debugging tool.

Javascript

This spreadsheet is an analysis of artificial reef structure that could form the basis for reef modelling in other languages. I know how to build such a model in Javascript, which would be ideal if we wanted to build a web site for it or to do more sophisticated graphics.

QuickTime™ and a
TIFF (Uncompressed) decompre
are needed to see this picture

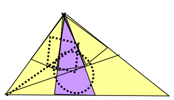
http://www.fablabdc.org/

FabLab

The ultimate version of this reef analysis could be instructions for fabricating custom-made reefs. We would need to get involved with FabLabDC or FabLab in San Diego.

New shapes

I have infinitely many nearly regular shapes that Archimedes should have discovered. No one seems to have recognized them since. Publishing these shapes is one of my top goals. Some of the shapes have religious significance and could be used in artificial reef memorials. I would use them to build a FabLab system for creating forms for such memorials.



Trisector

This is just playing with Geometry. A tool constructed with compass and straight edge can trisect an angle, easily for smaller angles.